Boundary-value problems with possibly discontinuous nonlinearities: a fixed point approach

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ABSTRACT: Our aim is to study some classes of boundary-value problems via Tychonov's like fixed-point results. To do this, in the first chapter, we state both classical fixed-point theorems and a simple but meaningful generalization obtained by O. Arino, S. Gautier, J.P. Penot, in 1984. Moreover, a version of the Schauder's theorem for discontinuous operators is reported, a recent result by R. López Pouso (2012).

Chapter 2 deals with the Dirichlet problem for a nonlinear, second-order differential equation. The first step is to get precise estimates for the corresponding Green function. Then, an existence theorem is obtained under Carathéodory conditions for the nonlinear term and special cases are next pointed out.

In the third chapter we focus on a boundary-value problem with discontinuous nonlinearities, where set-valued analysis plays an important role. After some preliminary results, a first existence theorem is presented. Recently, a different approach, which makes use of Schauder's theorem for discontinuous operators by López Pouso, has been pointed out. It allows us to find existence of solutions for wide classes of problems, by introducing the notion of viable and inviable admissible discontinuity curves. Two concrete examples ad applications of abstract results are made. Finally, a comparison between various results as well as possible extensions are performed.